

AD A060990

DDC FILE COPY

LEVEL

B.S. 2

A-1173  
AN ALGEBRAIC MODEL FOR CONTINUOUS-WAVE  
THERMAL-BLOWING EFFECTS

by

J. L. Brown

Ballistic Research Laboratory  
Aberdeen Proving Ground, Maryland

W. Green and R. Roberts

U. S. Army 52nd Infantry Center Systems Project Office  
Redstone Arsenal, Alabama

C. R. Wilkey

The Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts


July 1976

DDC  
PRELIMINARY  
EXPERIMENTAL  
RESULTS



The Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts 02139

Approved for public release; distribution unlimited

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT CATALOG NUMBER <b>(14) R-1173</b>
4. TITLE (and Subtitle) <b>(6) AN ALGEBRAIC MODEL FOR CONTINUOUS-WAVE THERMAL-BLOOMING EFFECTS,</b>		5. TYPE OF REPORT & PERIOD COVERED <b>(9) Technical Report</b>
7. AUTHOR(s) <b>(10) H./Breau, W./Evers, R./Sepucha &amp; C./Whitney</b>		6. PERFORMING ORG. REPORT NUMBER R-1173
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Charles Stark Draper Laboratory, Inc. Cambridge, Massachusetts 02139		8. CONTRACT OR GRANT NUMBER(s) <b>(15) DAAK40-78-C-0117 DAAK40-76-C-1145</b>
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Missile R&D Command High Energy Laser Systems Project Office Redstone Arsenal, Alabama 35809		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE <b>(11) July 1978</b>
		13. NUMBER OF PAGES 30 <b>(12) 31p.</b>
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		16a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) High-Energy Lasers Continuous-Wave Thermal Blooming HEL Propagation HEL System Analysis		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper presents an algebraic model for continuous-wave thermal-blooming effects that is accurate enough to well represent a large wave-optics data base, simple enough to suggest some previously unnoticed universal relationships, and complete enough to allow variation of the many physical variables of interest in systems analysis exercises. 		

408386

APPROVED BY	
DATE	DATE
DATE	DATE
DISTRIBUTION/AVAILABILITY	
DATE	
DATE	
DATE	

R-1173

AN ALGEBRAIC MODEL  
FOR CONTINUOUS-WAVE  
THERMAL-BLOOMING EFFECTS

by

H. Breaux  
Ballistic Research Laboratory  
Aberdeen Proving Ground, Maryland

W. Evers and R. Sepucha  
U.S. Army High Energy Laser Systems Project Office  
Redstone Arsenal, Alabama

C. Whitney  
The Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts

July 1978

Approved: *David B. Hoge*

10/13/78

The Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts 02139

#### ACKNOWLEDGMENT

This report was prepared by The Charles Stark Draper Laboratory, Inc. under Contracts DAAK40-76-C-1145 and DAAK40-78-C-0117 with the U.S. Army High Energy Laser Systems Project Office, Huntsville, Alabama.

Publication of this report does not constitute approval by the U.S. Army of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.

## TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
1 INTRODUCTION.....	1
2 INTENSITY DEGRADATION AS A FUNCTION OF POWER.....	4
3 PHYSICAL MODEL.....	8
4 PHASE INTEGRAL.....	13
5 CORRELATION DATA.....	17
6 SUMMARY OF FORMULAS FOR SYSTEMS ANALYSIS.....	23
LIST OF REFERENCES.....	26

## SECTION 1

### INTRODUCTION

The purpose of this document is to provide a standard algebraic model for a continuous-wave (CW) high-energy-laser (HEL) beam on target. The beam is degraded by thermal blooming as well as numerous other effects. The algebraic model is intended for systems analysis exercises, where the numerous parameters are to be explored, making more detailed computer simulation of atmospheric propagation impractical. It is an outgrowth of efforts to fit scaling laws to a large body of data generated by detailed atmospheric propagation simulations using a finite-difference wave-optics code.\*

Any such wave-optics code provides a detailed intensity profile on target, from which several different summary characteristics can be extracted. These include, for instance, peak intensity, line-of-sight beam dispersion, and beam area. The last of these further admits several definitions, including area to some percentage of total power, and area defined by a ratio of squared integral of intensity to integral of intensity squared (suggested independently by Lincoln Laboratory and Draper Laboratory researchers). If beams on target were Gaussian in shape, all such characteristics would convey equivalent information. The simplified algebraic model assumes that this is nearly the case, and speaks nominally of peak intensity on target.

Regression analysis of the results has shown that peak intensity can be correlated with an integral  $\Psi_h$ , which represents the accumulation along range (starting from the center of the aperture) of phase perturbation due to heating in a beam with absorption, scattering, convective clearing, and focusing. Figures 1, 2, and 3 show the tightness of the correlation obtained for three different beam shapes. The ordinate is the phase integral  $\Psi_h$  and the abscissa is the ratio  $R = (I_U - I)/I$ , where  $I$  is peak intensity and  $I_U$  is unbloomed peak intensity.

---

\* Data provided by D. Cordray of Naval Research Lab (NRL).

Any one of many numerically similar functional forms could be fit to the curves defined by the correlation data. This document selects one that has been found most useful because of its algebraic simplicity. Section 2 shows how the simple algebraic form leads to a number of universal relationships that are independent of almost all physical details of the propagation process. These relationships describe the variation of peak intensity as a function of power alone, with all other variables held constant. Important relationships are shown to depend only on beam shape. This conclusion has not previously been evident from other scaling laws.

Section 3 discusses the parameters appearing in Section 2, showing how each depends on actual physical variables that describe the engagement, the laser, and the atmosphere. Section 4 provides a technique for accurately evaluating the phase integral along range required in Section 3. For several common beam shapes, Section 5 discusses numerical values of regression parameters defined in Section 3. Section 6 provides a concise summary of all formulas required for systems analysis exercises.

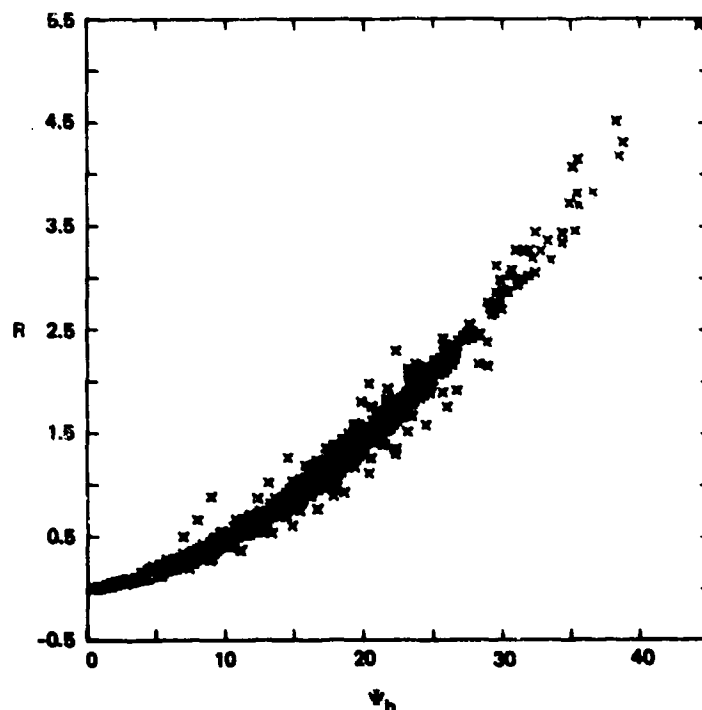


Figure 1. Correlation for infinite Gaussian beam.

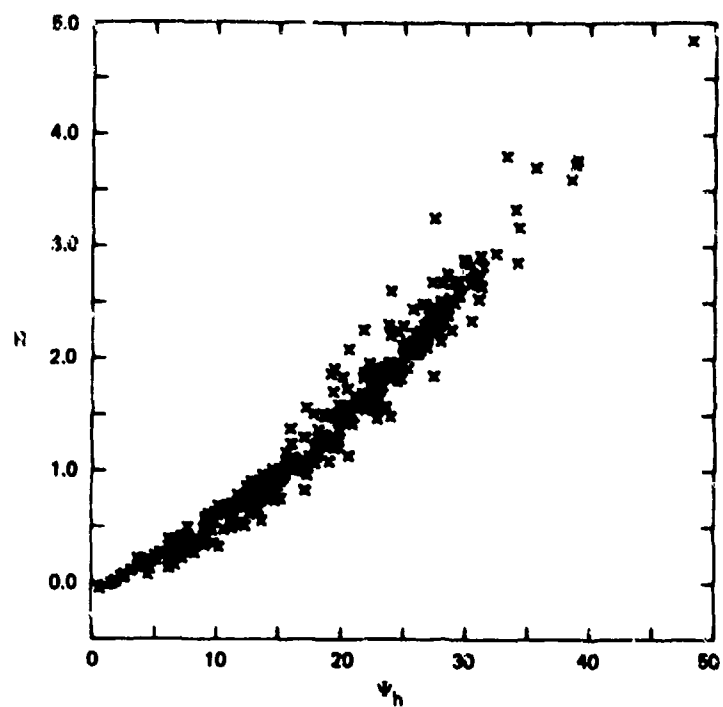


Figure 2. Correlation for  $1/e^2$  truncated Gaussian beam.

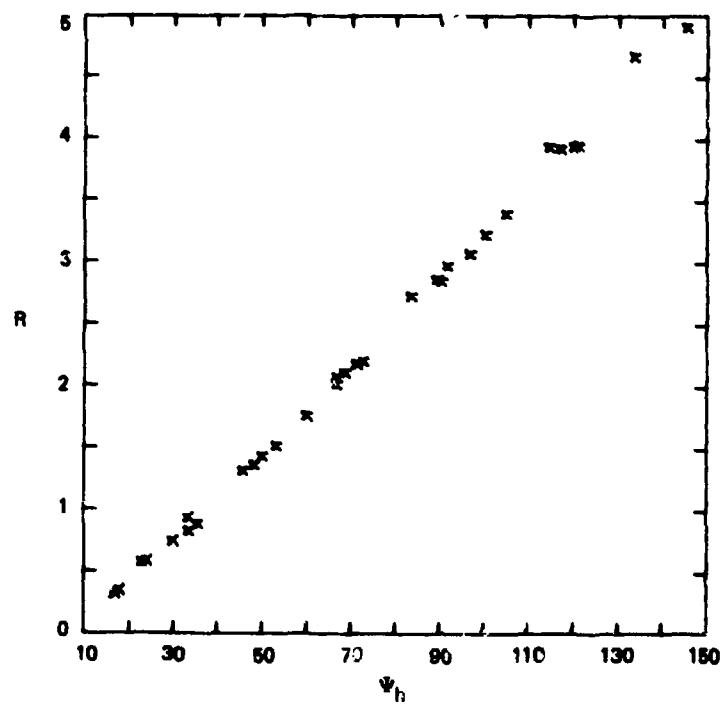


Figure 3. Correlation for uniform beam.



## SECTION 2

### INTENSITY DEGRADATION AS A FUNCTION OF POWER

For a class of problems differing only in time-average power ( $P$ ), the ratio of bloomed to unbloomed intensity focused on target can be modeled as

$$\frac{I(P)}{I_U(P)} = \frac{\sigma_L^2}{\sigma_L^2 + \sigma_B^2}$$

In this expression,  $\sigma_L^2$  is the 1/e line-of-sight beam dispersion in radians due to linear effects, including diffraction, beam quality, turbulence, and jitter. The latter two effects separate into "high" and "low" frequencies, which, respectively, do and do not impact blooming. The latter is represented in the term  $\sigma_B^2$ . To some extent, the form of  $I(P)/I_U(P)$  expresses the familiar idea of "rss-ing" (root-sum-squaring). For a Gaussian beam, dispersive effects combine by summing variances, and the denominator ( $\sigma_L^2 + \sigma_B^2$ ) resembles such a sum.

The blooming term ( $\sigma_B^2$ ) depends on power ( $P$ ) in a way that can be modeled by a variety of functional forms. The choice of form is a tradeoff between simplicity and range of validity. In this report, we use the simplest form known to be valid for power levels of practical interest. This form is nominally\*

$$\sigma_B^2 = C_B P^a$$

The use of  $P^a$  with  $a > 1$  allows reproduction of a well-known physical phenomenon: there exists a critical power  $P_C$  such that the

---

\* This type of formula has also been suggested and used by F. Gothardt and J. Wallace. In particular, Wallace suggested  $a = 3/2$ .

intensity on target is maximum. That  $P_C$  can be related to  $\sigma_L^2$ ,  $C_B$ , and  $a$  as follows. The unbloomed intensity on target is proportional to  $P$ , so that actual intensity is proportional to  $P[I(P)/I_U(P)]$ , or  $P\sigma_L^2/(\sigma_L^2 + C_B P^a)$ . Differentiating with respect to  $P$  and setting the derivative to zero gives

$$P_C = \left[ \frac{\sigma_L^2}{(a-1)C_B} \right]^{1/a}$$

From the value of  $P_C$ , the ratio  $I(P_C)/I_U(P_C)$  is readily found to be

$$\frac{I(P_C)}{I_U(P_C)} = \frac{a-1}{a}$$

This result is interesting because it depends only on the parameter  $a$ , expressive of beam shape alone, and not on  $\sigma_L^2$  or  $C_B$ , which contain much of the physics of the problem. It therefore presents a physical phenomenon that is essentially separable from other physical phenomena in the overall propagation process.

The result concerning  $I(P_C)/I_U(P_C)$  indicates what to expect from adaptive phase correction for thermal blooming. Since phase correction does not change beam shape, it will move the whole curve of intensity-out versus power-in in such a way that the new peak, the old peak, and the origin lie on a straight line, as illustrated in Figure 4.

Clearly,  $P_C$  marks the upper limit of power levels having practical interest. In fact, operation well below  $P_C$  may be of interest, so let us consider  $P$  at some fraction of  $P_C$ :  $P = P_C/b$ . Then

$$\frac{I\left(\frac{P_C}{b}\right)}{I(P_C)} = \frac{\sigma_L^2 + C_B (P_C)^a}{b \left[ \sigma_L^2 + C_B \left(\frac{P_C}{b}\right)^a \right]}$$

which simplifies to

$$\frac{I\left(\frac{P_C}{b}\right)}{I(P_C)} = \frac{ab^{a-1}}{b^a(a-1) + 1}$$

This result, too, is independent of  $\sigma_L^2$  and  $C_B$ , and therefore is independent of the phenomena controlling them.

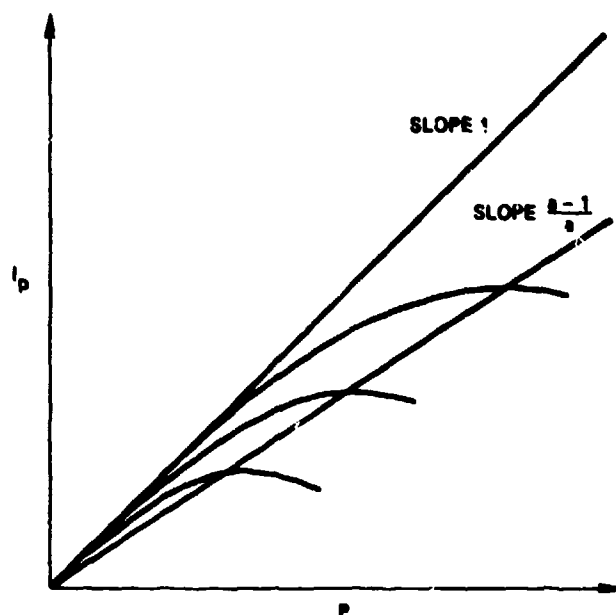


Figure 4. Adaptive correction of phase without change of beam shape.

The formula describing operation below  $P_C$  is plotted in Figure 5 for several typical values of  $a$ . For all cases, there is little point in operating at power levels above  $P_C/2$ , because more than 85% of the limiting peak intensity is already available at  $P_C/2$ .

The formulas for  $I(P_C)/I_U(P_C)$  and  $I(P_C/b)/I(P_C)$  can be combined to relate  $I(P_C/b)$  to  $I_U(P_C)$ . The result is

$$\frac{I\left(\frac{P_C}{b}\right)}{I_U(P_C)} = \frac{(a-1)b^{a-1}}{b^a(a-1) + 1}$$

This result, too, is independent of the parameters  $\sigma_L^2$  and  $C_B$  that carry most of the physics of the problem. It means essentially that if  $I_U(P_C)$  is specified,  $I$  for any other condition is implicitly specified.

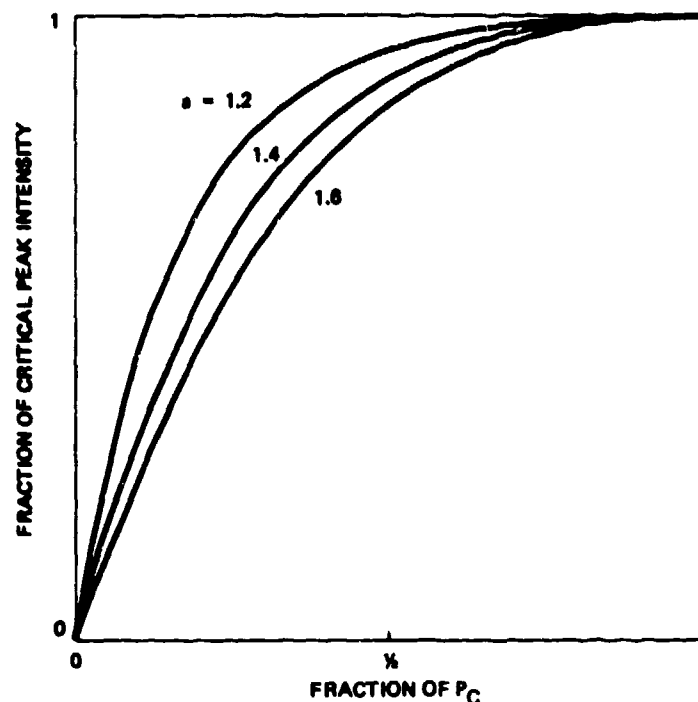


Figure 5. Operation below  $P_C$ .

The model presented in this section is most accurate in the regime of physical interest for systems modeling, namely, below the zero-wander value of  $P_C$ . This is the case because below this power there is relatively little blooming, with little attendant beam distortion and no beam breakup. There is then a one-to-one Gaussian-like correspondence between intensity  $I$  and the various dispersive  $\sigma^2$  terms. Operation far above such a power level requires a more complex model, involving linear and quadratic laser power terms in  $\sigma_B^2$ , presented by Breaux.<sup>(1)\*</sup>

The ratio-type results of this section are of a very general nature, and are valid regardless of the methodology used to evaluate  $a$ ,  $\sigma_L^2$ ,  $C_B$ , or  $I_U(P_C)$ . Specific techniques for evaluating these are provided in the Sections 3 through 5.

\* Superscript numerals refer to similarly numbered items in the List of References.

### SECTION 3

#### PHYSICAL MODEL

The model for degradation of intensity focused on target as a function of power has only three parameters ( $a$ ,  $\sigma_L^2$ , and  $C_B$ ), and requires only  $I_U(P_C)$  to specify intensity under any conditions. A small amount of data for a given class of problems differing only in  $P$  will suffice to determine all the required quantities. But extrapolating the results to any other class of problems requires some kind of physical model. The exponent  $a$  can be assumed to be independent of many physical variables, so the more pressing problems are to admit defocus, and to model  $\sigma_L^2$ ,  $C_B$ , and  $I_U(P_C)$ .

The physical variables affecting HEL propagation are designated in Table 1. In cases where the propagation path extends through significantly different altitudes, path averaging of parameters is required. (See Reference 1, p. 59 for applicable techniques regarding  $C_n^2$ .)

These parameters can be summarized in terms of four dimensionless numbers:

$$\text{Absorption number} \quad N_A = \alpha Z_t$$

$$\text{Slew number}^* \quad N_S = \frac{U_t}{U_w}$$

$$\text{Fresnel number} \quad N_F = \frac{2\pi R_m^2}{\lambda Z_t}$$

$$\text{Distortion number}^{*\dagger} \quad N_D = -2\sqrt{2} \left( \frac{\partial n}{\partial T} \right) \left( \frac{P Z_t N_A}{R_m^3 U_w C_P \rho_\infty n_\infty} \right)$$

\* Defined appropriate to this model.

† Subscript  $\infty$  means unperturbed natural value.

Table 1. Physical variables affecting  
HEL propagation.

Parameter	Variable	Units
<u>Engagement</u>		
Target range	$Z_t$	meters
Transverse target velocity	$U_t$	meters/second
<u>Laser</u>		
Aperture radius	$R_m$	meters
Time average power	$P$	Joules/second
Wavelength	$\lambda$	meters
Beam quality	$M$	no dimension
Beam shape		no dimension, not a scalar
<u>Atmosphere</u>		
Wind velocity	$U_w$	meters/second
Turbulence	$C_n^2$	meters <sup>-2/3</sup>
Absorption	$\alpha$	meters <sup>-1</sup>
Scattering	$\sigma$	meters <sup>-1</sup>
Index of refraction	$n$	no dimension
Temperature	$T$	degrees
Refraction gradient	$\partial n / \partial T$	degrees <sup>-1</sup>
Density	$\rho$	kilogram-meters <sup>-3</sup>
Heat capacity	$C_p$	Joules/degree-kilogram

There are also effective beam qualities  $N_Q$  and  $N'_Q$ , which represent the actual spreading of the beam due to various effects. These effects include beam shape and the blooming itself, and so  $N_Q$  and  $N'_Q$  must be defined later. A procedure is provided in Section 6.

The  $\sigma_L^2$  is 1/e radial beam dispersion due to linear effects. It includes a number of contributors, distinguished by different subscripts: diffraction and beam quality (D), high-frequency beam distortion and motion due to turbulence (T) and jitter (J), low-frequency beam wander (W) due to turbulence, jitter, and pointer-tracker effects. Thus, we have

$$\sigma_L^2 = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2$$

The formula for the diffraction contributor is

$$\sigma_D^2 = 0.5 \left( \frac{m' M \lambda}{2 R_m} \right)^2$$

where  $m'$  is characteristic of beam shape. Requiring  $\sqrt{2} \sigma_D$  to be the 63% beam radius on target makes  $m'$  equal to  $2/\pi$  for an infinite Gaussian beam, equal to 0.9166 for a  $1/e^2$  truncated Gaussian beam, and equal to 0.9202 for a uniform beam (see Reference 2).

The turbulence contributor refers specifically to the short-term high-frequency part of total turbulence. It has been found by Breaux<sup>(3)</sup> that for a large variety of beam shapes, total turbulence is well represented by

$$\sigma_{TT}^2 = \left( \frac{\sigma_D}{M} \right)^2 \left( \frac{D_e}{r_0} \right)^2$$

where  $r_0$  is the Fried coherence length for wave number  $k = 2\pi/\lambda$

$$r_0 = 2.10 \left[ 1.45 k^2 \int_0^z C_n^2(z) \left( \frac{z_t - z}{z_t} \right)^{5/3} dz \right]^{-3/5}$$

and  $D_e$  is an effective aperture size, appropriate to the beam shape. For infinite Gaussian, truncated Gaussian, or uniform beams, respectively, it is

$$D_e^2 = 8R_m^2, 3.7R_m^2, 4R_m^2$$

The short-term part ( $\sigma_T^2$ ) is smaller, varying between the values

$$\sigma_T^2 = 0.182 \left( \frac{\sigma_D}{M} \right)^2 \left( \frac{D_e}{r_0} \right)^2$$

for  $D_e/r_0 < 3$  and

$$\sigma_T^2 = \left( \frac{\sigma_D}{M} \right)^2 \left[ \left( \frac{D_e}{r_0} \right)^2 - 1.18 \left( \frac{D_e}{r_0} \right)^{5/3} \right]$$

for  $D_e/r_0 > 3$ .

To ensure model validity, the high-frequency turbulence should be limited to values small enough to cause no speckling, say  $\sigma_T < 2\sigma_D$ . The difference ( $\sigma_{TT}^2 - \sigma_T^2$ ) may or may not appear in the wander term  $\sigma_W^2$ , depending on the particular hardware implementation being modeled. Additional jitter and wander contributors arise from the particular hardware considered, and must be set by the user.

The dispersion due to blooming is to be combined with the linear dispersions simplified by  $\sigma_D^2$ ,  $\sigma_T^2$ ,  $\sigma_J^2$ , and  $\sigma_W^2$ . We proceed now to consider the blooming term

$$\sigma_B^2 = C_B P^a$$

The physical phenomenon that causes  $\sigma_B^2$  is accumulation along range of phase due to heating, which increases with power (P), and decreases, to some extent, with that portion of  $\sigma_L^2$  that actually experiences the blooming,



namely  $\sigma_L^2 - \sigma_W^2$ . It has been found possible to define a measure  $\Psi_h$  for heating phase such that the following expanded expression well represents  $\sigma_B^2$

$$\sigma_B^2 = C'_B (\sigma_L^2 - \sigma_W^2) (\Psi_h)^a$$

where  $C'_B$  is a dimensionless coefficient that depends on beam shape. Clearly

$$C_B = C'_B (\sigma_L^2 - \sigma_W^2) \left( \frac{\Psi_h}{P} \right)^a$$

The form of  $\Psi_h$  that makes the above representation of  $\sigma_B^2$  possible is proportional to  $N_D N_F / N_Q'$ . The phase integral  $(\Psi_h)$  also has an additional factor to make it into an integral along range that takes account of extinction, clearing, and focusing of the beam. Its evaluation is the subject of Section 4.

Next,  $I_U(P_C)$  is easily estimated by considering total extinction ( $\epsilon = \alpha + \sigma$ ), beam shape and quality, and the spreading due to linear effects. The estimate is

$$I_U(P_C) = \frac{P_C \exp(-\epsilon Z_t)}{2\pi Z_t^2 \sigma_L^2 B}$$

Here,  $B$  is a beam-shape factor, which can be roughly interpreted as the ratio of average intensity to "peak" intensity in the focal plane. For a Gaussian beam, it is exactly 1/2, and for any realistic beam shape, it is quite close to 1/2.

## SECTION 4

### PHASE INTEGRAL

In Section 3, the problem of evaluating  $C_B$  was reduced to the problem of evaluating the phase integral  $\Psi_h$ . Many integration procedures have been investigated, (1,4,5) and most can be made numerically adequate. The following is straightforward to explain. Let

$$\Psi_h = \int_0^1 \Psi_h(z) dz$$

where

$$z = \frac{z}{z_t}$$

$$\Psi_h(z) = \frac{N_D N_F \exp(-\epsilon z) \int dt I[x_0(z, t), y_0(z, t)]}{N_Q \left[ \frac{R_s(z)}{R_m} \right]^2 \left( \frac{2R_m}{U_w} \right) \left( \frac{P}{\pi R_m^2} \right)}$$

Here,  $\exp(-\epsilon z)$  represents extinction. The variable  $R_s(z)$  represents spot radius at  $z$ , approximated by rssiing focus and diffraction effects.

$$R_s(z) = R_m \left[ (1 - z)^2 + \left( \frac{2zN_Q}{N_F} \right)^2 \right]^{1/2}$$

The term  $2R_m/U_w$  represents the clearing time at the aperture, and provides a normalization for the time integration.  $P/\pi R_m^2$  provides a normalization for the intensity, which depends on time ( $t$ ) through  $x_0$  and  $y_0$ , which are both 0 for  $t = 0$ . For a beam slewed in the  $x$  direction

$$y_0(z, t) = 0$$

but  $x_0(z,t)$  varies with the local clearing velocity. Assuming  $x_0, y_0$  nondimensionalized by aperture radius  $R_m$

$$x_0(z,t) = \frac{-tU_\infty(z)}{R_s(z)}$$

with  $U_\infty(z)$  representing that velocity, approximated by

$$U_\infty(z) = U_w(1 + zN_S)$$

As an example, consider a Gaussian beam with amplitude

$$A_0(x_0, y_0) = \exp(-x_0^2 - y_0^2)$$

For the Gaussian profile, the normalized intensity

$$\frac{I[x_0(z,t), y_0(z,t)]}{(P/\pi R_m^2)} = 2 \exp\left[-2\left(\frac{tU_\infty(z)}{R_s(z)}\right)^2\right]$$

The time integral of normalized intensity is

$$\left[\frac{2R_s(z)}{U_\infty(z)}\right] \left(\frac{U_w}{2R_m}\right) \sqrt{\frac{\pi}{2}} \left(\frac{1}{2}\right)$$

Absorbing the  $\sqrt{\pi/2} (1/2)$  in the overall  $C_B'$  relating to beam shape, and substituting for  $U_\infty(z)$  and  $R_s(z)$ , leaves the required  $z$  integral to be just

$$\eta_h = \int_0^1 \frac{N_D N_F \exp(-\epsilon z) dz}{N_Q'(1 + zN_S) \left[ (1-z)^2 + \left(\frac{2zN_Q}{N_F}\right)^2 \right]^{1/2}}$$

The integral does not appear to be amenable to direct analytic evaluation, so the options are numerical integration or analytic approximation. Typically, numerical integration is difficult because

small steps are required to handle the rapid variation of the integrand. Therefore, analytic approximation is preferred. Probably the fewest approximations are required if integration by parts is employed. In that case we find

$$\Psi_h = \left[ \exp(-\epsilon z) \Psi_h' \right]_0^1 + \epsilon z_t \int_0^1 \exp(-\epsilon z) \Psi_h' dz$$

The indefinite integral

$$\Psi_h' = \int \left( \frac{N_D N_F}{N_Q} dz \right) / (1 + z N_S) \left[ (1 - z)^2 + \left( \frac{2z N_Q}{N_F} \right)^2 \right]^{1/2}$$

is available in an integral table by Klerer and Grossman.<sup>(6)</sup> Naturally, the expression contains an integration constant. In principle, this constant needs to be reset in such a way as to nullify the remainder term

$$\epsilon z_t \int_0^1 \exp(-\epsilon z) \Psi_h' dz$$

This can be accomplished at least approximately by using  $\Psi_h'(z) - \Psi_h'(1/2)$  in place of  $\Psi_h'(z)$ . This makes the expression for  $\Psi_h$  quite complicated. Utility is greatly served by reducing it to the case of small attenuation  $\epsilon z_t$ , where the integration constant has no impact, and  $\Psi_h$  reduces to

$$\Psi_h = \left[ \frac{N_D N_F}{N_Q A} \right] \exp \left[ \frac{-\epsilon z_t}{2} \right] \ln \left| \frac{(N_S + 1 + A)(N_S + 1)}{-B + N_S + 1 + AC} \right|$$

where

$$\begin{aligned} A &= [(N_S + 1)^2 + C^2]^{1/2} \\ B &= N_S + 1 + C^2 \\ C &= \frac{2N_Q}{N_F} \end{aligned}$$

The condition for this result to be valid can be stated as

$$1 - \exp(-\varepsilon z_c) \psi'_h\left(\frac{1}{2}\right) \ll -\psi'_f(0) + \exp(-\varepsilon z_c) \psi'_h(1)$$

## SECTION 5

### CORRELATION DATA

Extensive simulations of atmospheric propagation have been performed at NRL using a finite-difference wave-optics code, and these provide the data base that establishes a strong correlation of the form suggested in this report relating intensity on target to the parameter  $\Psi_h$ . There is very little scatter, and that which does exist may be attributed to one of two factors:

- (1) Earlier correlation studies by Zeiders<sup>(7)</sup> established that the spot size in the integrand denominator of  $\Psi_h$  should not be just the ideal diffraction-limited value, mainly because of the iterative effect of blooming upon itself. To account somewhat for this in a way that maintains reasonable simplicity, a free constant (m) scaling the diffraction spot was introduced and evaluated by minimizing residuals. Actually, such scaling must depend to some extent on physical variables, especially power. The idea of iteration to admit variable m has been tried, but so far has not been sufficiently successful to justify the effort.
- (2) The raw data generated were not all of the same form. Data for the infinite Gaussian beam comprised peak intensities, whereas those for the truncated Gaussian comprised  $1/e^2$  area, and those for the uniform beam comprised the more complex area measure of the form

$$A = \frac{\left[ \int I \, dx \, dy \right]^2}{\int x^2 \, dx \, dy}$$

The latter functional (A) is currently thought to lead to the tightest correlations.

The results of analyzing three beam shapes—infinite Gaussian,  $1/e^2$  truncated Gaussian, and uniform beam—are presented in Tables 2 through 4 and plotted in Figures 6 through 8. Supposing that  $N_D$  and  $N_F$  are fixed and  $\sigma_W^2$  equals zero, let us compare the three beam types. We recall that  $C_B$  is proportional to  $C_B' \sigma_L^2$  and that

$$P_C = \left[ \frac{\sigma_L^2}{(a-1)C_B} \right]^{1/a}$$

and

$$\frac{I(P_C)}{I_U(P_C)} = \frac{a-1}{a}$$

The results tend to follow intuitively understandable patterns. First, the parameter  $m$  affects the tightness of the correlation much less in the case of the Gaussian beams than in the case of the uniform beam. This is to be expected because blooming in the Gaussian case is driven mainly by gradients near the aperture, whereas for the uniform beam such gradients are minimized and blooming is driven by beam shape nearer the focus, which is described by  $m$ . Secondly, the uniform beam has the smallest  $a$  and  $C_B'/\sigma_L^2$ , resulting in by far the largest  $P_C$ . The large  $P_C$  is to be expected because, with less gradient near the aperture, there is less lensing effect even at high power. The small  $a$  means, however, that the  $P_C$  region is very broad and  $I(P_C)/I_U(P_C)$  is very small. The two Gaussian beams have anomalously different  $C_B'$ ,  $a$ , and  $m$ , but they turn out to have similar small  $P_C$  values. But because  $a$  is larger for these cases,  $I(P_C)/I_U(P_C)$  is somewhat larger.

The actual peak intensity on target at  $P_C$  depends on  $P_C$ , on  $I(P_C)/I_U(P_C) = (a-1)/a$ , and on the diffraction-limited spot radius  $r_s$

$$I(F_C) = \frac{P_C}{\pi r_s^2} \left( \frac{a-1}{a} \right)$$

The spot radius for the three beam shapes, respectively, is proportional to  $2/\pi$ , 0.9166, and 0.9202, with the result that  $I(P_C)$  for the truncated Gaussian is only half that of the infinite Gaussian. The value for the uniform beam is nearly three times what it is for the truncated Gaussian, but at the price of nearly six times the power (which may nevertheless be acceptable).



Table 2. Infinite Gaussian:  $I(P)/I_U(P) = (1 + C_B^2 v^2)^{-1}$ .

	rms Error	Maximum Error	$C_B^2$	a	m
	0.142	0.949	0.011851	1.5029	1.0
	0.109	0.667	0.010796	1.5891	1.5
	0.102	0.614	0.010612	1.6189	1.75
Chosen fit	0.101	0.565	0.010590	1.6419	2.0
	0.112	0.607	0.010919	1.6715	2.5

$$P_C = 20.903 \quad I(P_C)/I_U(P_C) = 0.39035 \quad I(P_C) = 20.163$$

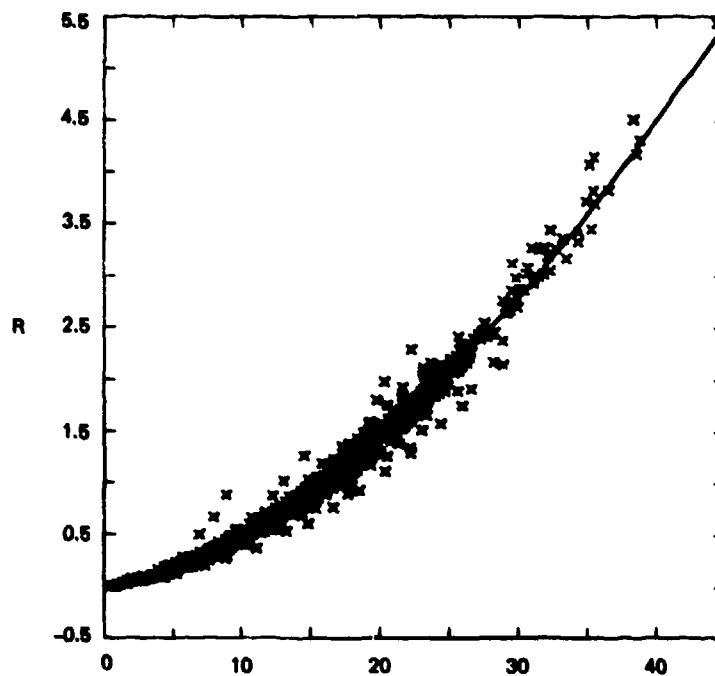


Figure 6. Functional fit for infinite Gaussian beam.

Table 3.  $1/e^2$  truncated Gaussian:  $I(P)/I_U(P) = (1 + C'_B \psi_h^a)^{-1}$ .

	rms Error	Maximum Error	$C'_B$	a	m
Chosen fit	0.170	0.981	0.018600	1.4323	3.5
	0.169	0.924	0.019023	1.4509	4.0
	0.172	0.874	0.019630	1.4646	4.5
	0.178	0.828	0.020398	1.4743	5.0
	0.187	0.788	0.021313	1.4805	5.5
	0.198	0.752	0.022366	1.4838	6.0

$$P_C \approx 27.780$$

$$I(P_C)/I_U(P_C) = 0.31077$$

$$I(P_C) \approx 10.276$$

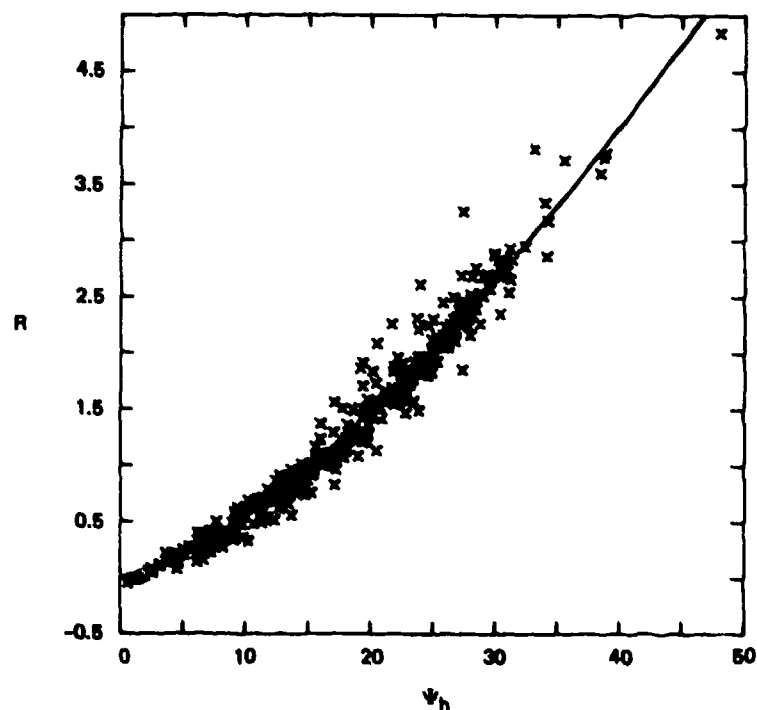


Figure 7. Functional fit for  $1/e^2$  truncated Gaussian beam.

Table 4. Uniform beam:  $I(P)/I_U(P) = (1 + C'_B \psi_h^a)^{-1}$ .

	rms Error	Maximum Error	$C'_B$	a	m
	0.092	0.204	0.014732	1.1380	0.5
	0.066	0.157	0.014395	0.1614	0.75
Chosen fit	0.063	0.139	0.014264	1.1777	1.0
	0.093	0.255	0.014335	1.1986	1.5
	0.112	0.317	0.014474	1.2054	1.75
	0.130	0.374	0.014660	1.2106	2.0

$$P_C \approx 160.09$$

$$I(P_C)/I_U(P_C) = 0.15089$$

$$I(P_C) \approx 28.526$$

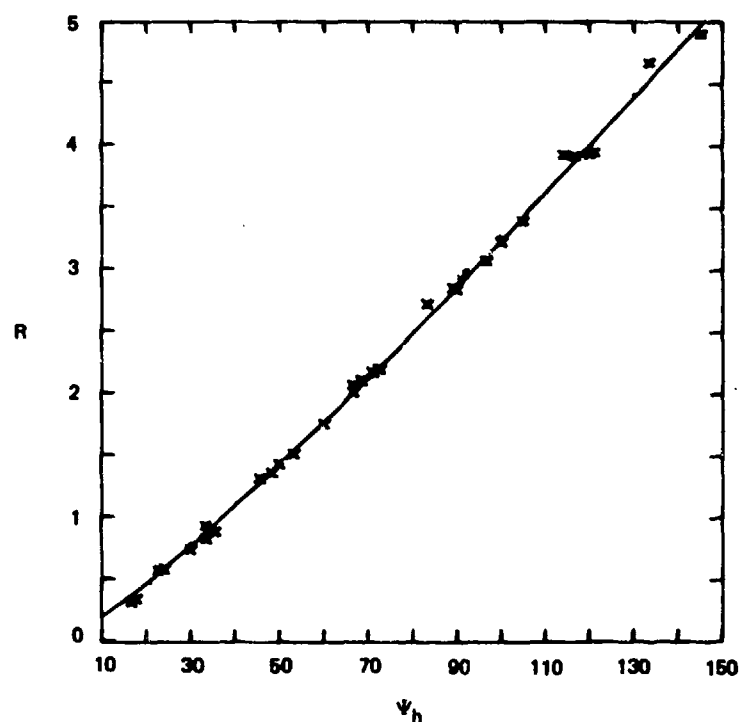


Figure 8. Functional fit for uniform beam.

## SECTION 6

### SUMMARY OF FORMULAS FOR SYSTEMS ANALYSIS

Suppose that one wishes to calculate peak intensity on target for a given set of conditions. The required steps are:

- (1) Choose beam shape and set  $C'_B$ ,  $a$ ,  $m$ ,  $m'$ , and  $m''$ .

	$C'_B$	$a$	$m$	$m'$	$m''$
Infinite Gaussian	0.010590	1.6419	2.0	0.6366	1
Truncated Gaussian	0.028727	1.3715	4.0	0.9166	0.8893
Uniform Beam	0.014264	1.1777	1.0	0.9202	1.124

- (2) Evaluate nondimensional numbers.

Absorption number  $N_A = \alpha Z_t$

Slew number  $N_S = \frac{U_t}{U_w}$

Fresnel number  $N_F = \frac{2\pi R_m^2}{\lambda Z_t}$

Distortion number  $N_D = (2.333 \times 10^{-9}) \frac{P Z_t N_A}{R_m^3 U_w}$

(3) Evaluate linear dispersions.

$$\sigma_{D0}^2 = 0.5 \left( \frac{\lambda}{\pi R_m} \right)^2$$

$$\sigma_D^2 = 0.5 \left( \frac{m' M \lambda}{2 R_m} \right)^2$$

$$k = \frac{2\pi}{\lambda}$$

$$r_0 = 3.017 (k^2 c_n^2 z_t)^{-3/5}$$

$$D_e^2 = 8R_m^2, 3.7R_m^2, \text{ or } 4R_m^2 \text{ for infinite Gaussian, } 1/e^2 \text{ truncated Gaussian, or uniform beam, respectively}$$

$$\sigma_{TT}^2 = \left( \frac{\sigma_D}{M} \right)^2 \left( \frac{D_e}{r_0} \right)^2$$

$$\sigma_T^2 = 0.182 \left( \frac{\sigma_D}{M} \right)^2 \left( \frac{D_e}{r_0} \right)^2 \text{ for } \frac{D_e}{r_0} < 3$$

$$\sigma_T^2 = \left( \frac{\sigma_D}{M} \right)^2 \left[ \left( \frac{D_e}{r_0} \right)^2 - 1.18 \left( \frac{D_e}{r_0} \right)^{5/3} \right] \text{ for } \frac{D_e}{r_0} > 3$$

$$\sigma_J^2 \text{ appropriate to hardware}$$

$$\sigma_W^2 \text{ appropriate to hardware}$$

$$\sigma_L^2 = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2$$

(4) Evaluate effective beam qualities.

$$N_Q = \frac{(m^2 \sigma_D^2 + \sigma_T^2 + \sigma_J^2)^{1/2}}{\sigma_{D0} m^n}$$

$$N'_Q = \frac{(\sigma_L^2 - \sigma_W^2)^{1/2}}{(\sigma_D/M)}$$

(5) Evaluate blooming dispersion.

$$\epsilon = \alpha + \sigma$$

$$C = \frac{2N_Q}{N_F}$$

$$B = N_S + 1 + C^2$$

$$A = [(N_S + 1)^2 + C^2]^{1/2}$$

$$\psi_h = \left[ \frac{N_D N_F}{N_Q A} \right] \exp \left[ \frac{-\epsilon Z_t}{2} \right] \ln \left| \frac{(N_S + 1 + A)(N_S + 1)}{-B + N_S + 1 + AC} \right|$$

$$\sigma_B^2 = C_B^2 (\sigma_L^2 - \sigma_W^2) (\psi_h)^a$$

(6) Evaluate peak intensity.

$$I(P) = \frac{P \exp(-\epsilon Z_t)}{\pi Z_t^2 (\sigma_L^2 + \sigma_B^2)}$$

#### LIST OF REFERENCES

1. Breaux, H.J., A Methodology for Development of Simple Scaling Laws for High Energy CW Laser Propagation, Ballistic Research Laboratory Technical Report ARBRL-TR-02039, January 1978.
2. Holmes, D.A., J.E. Korba, and P.V. Avizonis, "Parametric Study of Apertured Focused Gaussian Beams", Applied Optics, Vol. 11, p. 565, 1972.
3. Breaux, H.J., Correlation of Extended Huygens-Fresnel Turbulence Calculations for a General Class of Tilt Corrected and Uncorrected Laser Apertures, Ballistic Research Laboratory Interim Memorandum Report No. 600, May 1978.
4. Breaux, H.J., A Phase Integral Scaling Law Methodology for Combined Repetitive Pulse and CW High Energy Laser Propagation, Ballistic Research Laboratory Interim Memorandum Report, August 1977.
5. Wallace, J., and C. Whitney, A Simplified Propagation Code for Laser Systems Analysis, Charles Stark Draper Laboratory Report C-4487, June 1977.
6. Klerer, M., and F. Grossman, A New Table for Indefinite Integrals, Dover, 1971.
7. Zeiders, G.W., A Study of Wave Characteristics Influence on Laser Selection for Applications: Propagation Analysis, W.J. Schaefer Associates Report WJSA TR-74-18, August 1974.